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Performance Study of Odd Bit Modulation Schemes in OFDM Systems Under Combined Presence of AWGN, Phase Noise and Frequency Selective Fading

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Abstract. *Adaptive constellation Quadrature Amplitude Modulation (QAM) provides benefits for the transmitter during adaptation process of the constellation size in accordance with varying channel quality with optimal speed and required Bit Error Rate (BER). In order to improve adaptive QAM efficiency, the use of odd-bit cross-QAM constellations has been suggested as an advanced technology which improves transmission capabilities of modern multicarrier approaches, for example, of OFDMA (Orthogonal Frequency Division Multiple Access). The use of cross odd-bit QAM technology is more effective because of the fact that cross odd-bit QAM systems outperform conventional systems in terms of the effectiveness of their performance rate and in terms of peak-to-average power ratio (PAPR). Cross odd-bit QAM constellations are therefore believed to be successful best of modulation systems for the cases when odd-bit modulation is necessary. However, despite the large amount of existing literature on odd-bit QAM systems, there has been little research conducted regarding the performance of OFDMA systems applied in the context of the use of odd-bit QAM formats under conditions of AWGN (Additive White Gaussian Noise), PhN (Phase Noise) and frequency-selective fading. The numerical results indicate that cross odd-bit QAM schemes yield pronounced BER gains at lower modulation orders, while offering only marginal improvements at higher orders. For instance, 8-QAM achieves the target performance with approximately 6 dB lower Eb/N0 than the 16-QAM system. Therefore, 8-QAM provides an Eb/N0 gain of 6 dB under the same error-rate condition. This indicates that 8-QAM is more power-efficient; however, 16-QAM provides a higher data rate because it carries more bits per symbol.*

Keywords : *Adaptive QAM, Cross Odd-Bit QAM, OFDMA, Bit Error Rate (BER), Peak-to-Average Power Ratio (PAPR)*

I. Introduction

Orthogonal frequency-division multiple access (OFDMA) extends OFDM by allocating subsets of orthogonal subcarriers and time-frequency resources to different users [21], [22]. It can be viewed as an extension of conventional OFDM in which the available subcarriers are divided into groups and flexibly assigned to different users. In this way, OFDMA preserves the main benefits of OFDM high spectral efficiency, robustness to inter-symbol interference, and good performance over multipath fading channels while adding the ability to allocate frequency-domain resources on a fine, per-user basis. By employing different arrangements of subcarriers for separate users operating together in real-time, OFDMA makes it possible to achieve simultaneous transfers by multiple users thus increasing the overall efficiency and shortening the time of transmission when multiple users are being served at once.

Current communication systems demand flexible technologies used for configuring the modulation which has to take into consideration such elements as the transfer rate, power consumption, bandwidth needed for the signal and robustness in the mean of its fidelity. In this context, the implementation of the odd-bit cross QAM within adaptive QAM systems is widely accepted as a means of increasing transfer speed while still keeping the same boundaries of the power consumption and the costs of operating the bandwidth. Hence, it becomes necessary to conduct the research of the performance of the BER of odd-QAM cross M-QAM technology in the presence of different types of real-world impairment.

II. Literature Review

The foundational error-performance assessment in depolarizing channels often utilizes the moment-generating function technique, which creates a common base for obtaining the average BER and SER over the various types of fading distribution [1],[2].

The models for determining the distinct BER for the case of cross-QAM in AWGN and Rayleigh fading indicate the reasons for the popularity of cross constellations, which are especially beneficial when the number of bits per symbol is odd: they provide better peak and average transmit power than the rectangular QAM [3]. Following this work, the exact SEP results for the case of 32-cross-QAM over Nakagami fading have been obtained [4] and the arbitrary-order cross-QAM with respect to different fading models has been studied [5] as well as the cross-QAM

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with maximum ratio combining through the consideration of generalized fading models [6]. The related studies provide the accurate Rayleigh-fading SEP analysis [7], results for the 128-cross-QAM and 512-cross-QAM cases [8] and simplified expressions based on the equivalent rectangular-QAM technology use [9]. The latest review provides a study of cross, rectangular, star, and hexagonal QAM methods and compares them with respect to energy efficiency, PAPR, mapping, SER, and BER results [10].

In the case of the OFDM technology, the phase noise occurring at oscillators destroys the orthogonality of subcarriers since it appears as CPE and ICI. The earliest studies have dealt with the analysis of the effects of CPE and ICI behavior and the performance of the oscillators [12] including studies that investigated the behavior of CPE and ICI [11]. The current research investigates CPE and ICI consequences in cases of frequency-selective Rician and Rayleigh fading channels [14] while the recent research on Wiener phase noise analysis in frequency-selective channels has been published [15] as well as CPE and ICI methods used for compensation of frequency-selective channel characteristics [16].

Adaptive M-QAM changes the constellation size and, where permitted, transmit power according to channel quality to improve average spectral efficiency under BER and power constraints [17]. This framework was extended to adaptive coded modulation [18], Nakagami fading [19], and unified rate-power-BER adaptation [20]. In OFDMA, such link adaptation is combined with subcarrier and power allocation; resource-allocation surveys categorize common rate-adaptive and margin-adaptive objectives [21], [22].

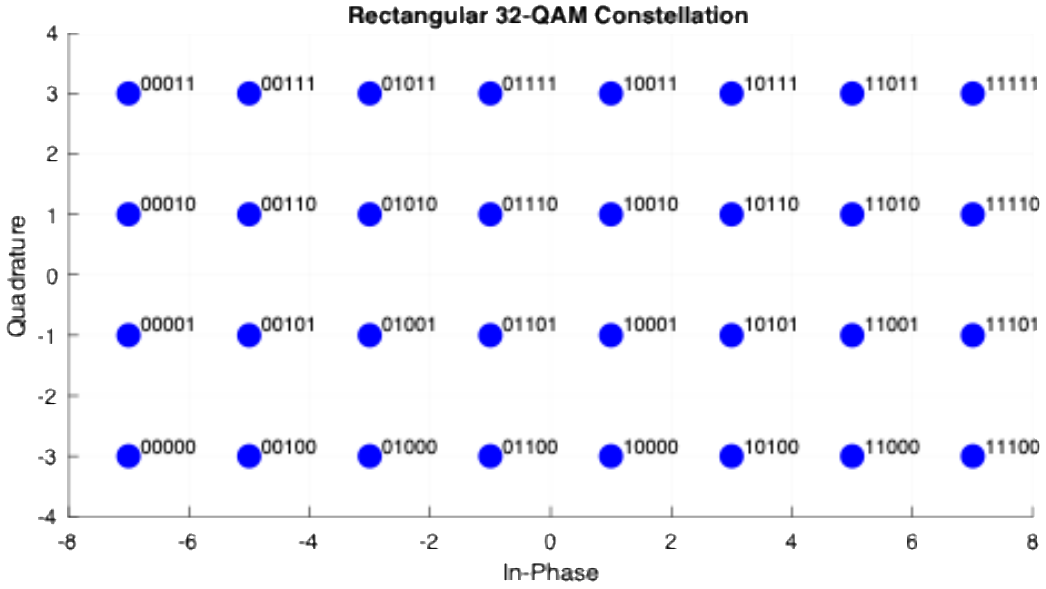
Frequency-selective multipath fading motivates multicarrier transmission and one-tap frequency-domain equalization when the cyclic prefix is sufficiently long [23], [24]. Standard digital-communications treatments provide the AWGN, fading, and M-QAM error-performance foundations used to interpret the simulations in this study [25].

Odd-Bit QAM Constellations

Even-bit QAM schemes such as 16-QAM, 64-QAM, and 256-QAM are commonly implemented as square constellations, whereas odd-bit modes are useful when intermediate spectral efficiencies such as 5, 7, or 9 bits per symbol are required [10], [25].

One way to implement odd-bit QAM is a rectangular constellation, such as the 32-QAM arrangement in Figure 2. Compared with cross-QAM at the same minimum distance, rectangular odd-bit constellations generally require greater peak and average energy [3], [5], [10].

Figure 2: 32 QAM (rectangular 32 QAM) constellation diagram

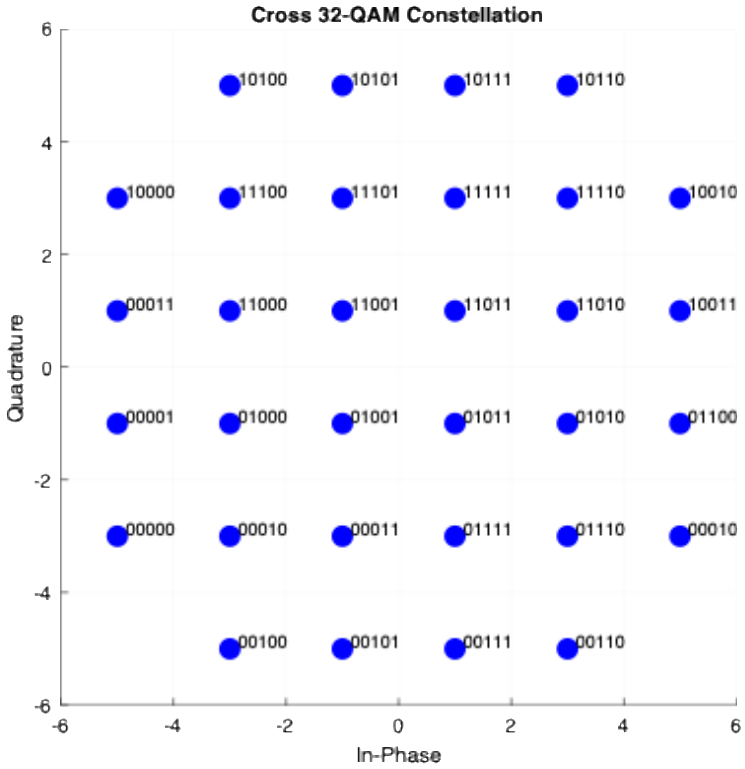


Cross-QAM achieves a less bulky arrangement with a fixed minimum Euclidean distance. The case of cross 32-QAM, depicted in Figure 3, circumvents the high-energy outer-corner locations from which its rectangular counterpart would have sprung.

Ultimately, the updated arrangement of constellation points benefits energy efficiency and results in improved performance based on the bit rate delivery (BER) and peak-to-average power ratio (PAPR). The following analysis will illustrate mathematically that cross 32-QAM would provide more energy efficiency than rectangular 32-QAM.

Figure 3: Cross 32-QAM Constellation

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The constellation points of an M-QAM modulation scheme are defined by their in-phase (x) and quadrature (y) components. The average energy of the signal is typically calculated based on these components.

$$E_{\text{av}}(\text{M-QAM}) = \mathbb{E}(x^2 + y^2) = \frac{1}{M}\sum_{i=1}^{M}(x_i^2 + y_i^2)$$

Using equation (1) to evaluate the average energy of both the rectangular 32-QAM (32-RQAM) and the cross 32-QAM (32-CQAM) constellations illustrated in Figures (2) and (3), the following results are obtained.

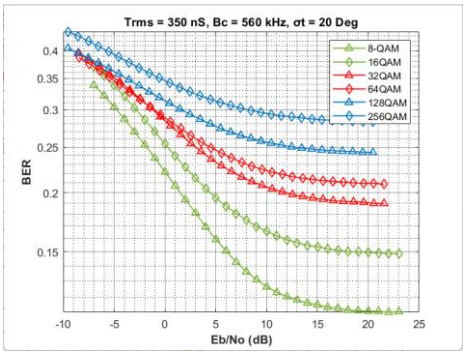
$$E_{\text{av}}(32-\text{RQAM}) = 26$$
$$E_{\text{av}}32-\text{CQAM}) = 20$$

Based on the calculated average energies and the resulting energy difference, 32-RQAM requires approximately 1.14 dB more average energy than 32-CQAM to maintain the same minimum Euclidean distance. This agrees with published cross-QAM analyses reporting an advantage of about 1.1 dB or more over rectangular QAM under comparable conditions [5], [6], [10].

Simulation Results

HTML adalah bahasa markup standar yang digunakan untuk membuat struktur dan konten sebuah halaman. HTML memberikan kerangka dasar yang memungkinkan berbagai elemen seperti teks, gambar, dan tautan ditampilkan pada halaman web. HTML merupakan komponen utama dalam pengembangan website, termasuk untuk sistem informasi acara, di mana struktur yang baik akan meningkatkan navigasi dan kenyamanan pengguna. [6]

Figure 4: BER and Eb/No Gains at σt = 20o



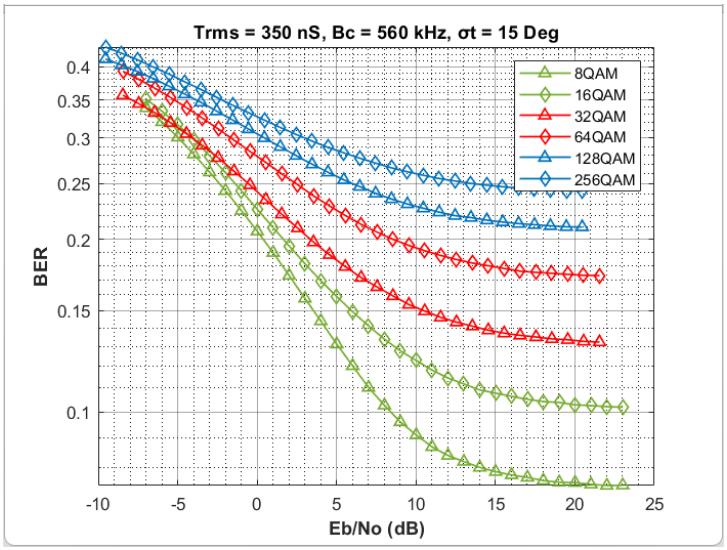
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The table 1 shows the Eb/No gain achieved by lower-order QAM schemes over higher-order QAM schemes at σt = 20o.

Table 1: Eb/No gain for lower-order QAM vs higher-order QAM schemes at σt = 20o.

Modulation Sizes	Common BER	Eb/No Gain
8-QAM vs 16 QAM	16×10^{-2}	6 dB
32-QAM vs 64 QAM	23×10^{-2}	3 dB
128-QAM vs 256 QAM	30×10^{-2}	6 dB

Figure 5: BER and Eb/No Gains at $\sigma_t = 15^o$



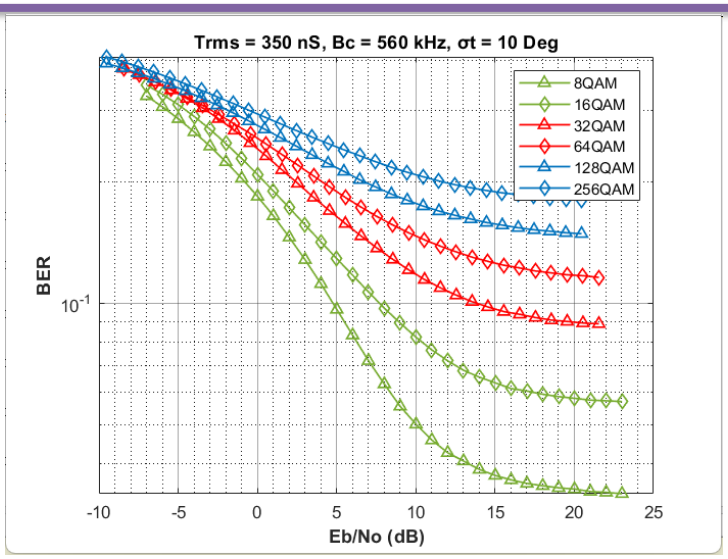
The table 2 shows the Eb/No gain achieved by lower-order QAM schemes over higher-order QAM schemes at $\sigma_t = 15^o$.

Table 2: Eb/No gain for lower-order QAM vs higher-order QAM schemes at $\sigma_t = 15^o$.

Modulation Sizes	Common BER	Eb/No Gain
8-QAM vs 16 QAM	12×10^{-2}	4 dB
32-QAM vs 64 QAM	20×10^{-2}	4 dB
128-QAM vs 256 QAM	26×10^{-2}	5 dB

Figure 6: BER and Eb/No Gains at σt = 10o

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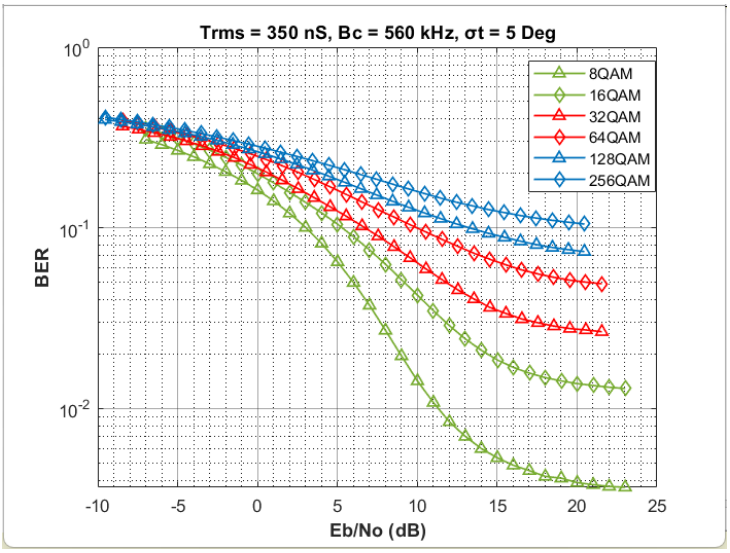


The table 3 shows the Eb/No gain achieved by lower-order QAM schemes over higher-order QAM schemes at σt = 10o.

Table 3: Eb/No gain for lower-order QAM vs higher-order QAM schemes at σt = 10o.

Modulation Sizes	Common BER	Eb/No Gain
8-QAM vs 16 QAM	6×10^{-2}	6 dB
32-QAM vs 64 QAM	13×10^{-2}	5 dB
128-QAM vs 256 QAM	19×10^{-2}	5 dB

Figure 7: BER and Eb/No Gains at σt = 5o



The table 4 shows the Eb/No gain achieved by lower-order QAM schemes over higher-order QAM schemes at σt = 5o.

Table 4: Eb/No gain for lower-order QAM vs higher-order QAM schemes at σt = 5o.

Modulation Sizes	Common BER	Eb/No Gain
8-QAM vs 16 QAM	15×10^{-3}	6 dB
32-QAM vs 64 QAM	6×10^{-2}	6 dB
128-QAM vs 256 QAM	11×10^{-2}	5 dB

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Phase-Noise Model

Unlike AWGN, phase noise is commonly modeled as a multiplicative rotation. The discrete received signal affected by both AWGN and phase noise is expressed as [11], [12], [14], [15]:

$$\boldsymbol{r_k = s_k . e^{j\emptyset_k} + n_k}$$

Where ∅_k represents the random phase noise sample at sample k.

In OFDM transceivers, phase noise appears principally as common phase error (CPE) and inter-carrier interference (ICI) [12]–[16]. In this study, the sample phase is represented by a two-component model:

$$\boldsymbol{\emptyset_k = \theta_k + \varphi_k}$$

where:

- The colored drift component is modeled as a Wiener process [11], [15]:

$$\boldsymbol{\theta_k = \theta_{k-1} + \Delta_k}$$

The innovation is modeled as an independent Gaussian random variable; this discrete-time Wiener process is a standard model for free-running oscillator phase noise [11], [15].

- The second term is modeled as a zero-mean stationary white Gaussian sequence independent of the drift term. Under this explicit modeling assumption, the total phase-noise variance is the sum of the component variances:

$$\boldsymbol{\sigma_t^2 = \sigma_\theta^2 + \sigma_\varphi^2}$$

Scope and Limitations

The results are uncoded and simulation-based. Their interpretation is limited by the absence of a complete reproducibility table specifying the OFDMA dimensions, cyclic-prefix length, subcarrier allocation, channel impulse response, equalizer, phase-noise parameters, and Monte Carlo stopping rule. Consequently, the reported numerical gains should not be generalized to coded links, other fading profiles, or other oscillator spectra without further validation.

III. Conclusion

The objective of the study was to examine the uncoded BER of an OFDMA system using cross odd-bit QAM constellations subject to the effects of AWGN, phase noise, and frequency selective fading. Results indicate that modulation order of the scheme plays major role in the trade-off between power effectiveness and spectral efficiency since lower order techniques are better in terms of BER performance, while larger order methods are more efficient in the number of messages sent per signal unit at the expense of the sensitivity to transmission conditions.

Moreover, the results seem to suggest the application of cross odd-bit QAM as the odd-bit configuration will become much more effective in terms of energy consumption than conventional rectangular odd bit QAM. Par example, the average energy of symbols shows that cross 32 QAM does not require as much energy as rectangular 32 QAM in order to deliver signal with the same minimum Euclidean distance. Such conclusions regarding the geometry and energy efficiency correspond to existing knowledge on cross QAM theories [3]–[6], [10].

Thus, cross odd-bit QAM may show maximum benefits for applications that require reliable delivery of signals with respect to energy saving at low and medium modulation orders. For adaptive modulation systems, one may use 8-QAM or 32-QAM depending on the channel conditions whereas larger formats should be used only in case of good channel conditions having high-level signals and effectively countering phase noise. Further studies may develop this framework by introducing channel encoding, channel estimation failures, phase compensation techniques, user scheduling, and adaptive modulation coding for wireless communication systems of 5G and onwards.

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